Effect of nonadiabaticity of dust charge variation on dust acoustic waves: Generation of dust acoustic shock waves

M. R. Gupta, Susmita Sarkar,* Samiran Ghosh, M. Debnath, and Manoranjan Khan
Centre For Plasma Studies, Faculty of Science Jadavpur University, Calcutta 700 032, India
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The effect of nonadiabaticity of dust charge variation arising due to small nonzero values of $\tau_{\rm ch}/\tau_d$ has been studied where $\tau_{\rm ch}$ and τ_d are the dust charging and dust hydrodynamical time scales on the nonlinear propagation of dust acoustic waves. Analytical investigation shows that the propagation of a small amplitude wave is governed by a Korteweg–de Vries (KdV) Burger equation. Notwithstanding the soliton decay, the ''soliton mass'' is conserved, but the dissipative term leads to the development of a noise tail. Nonadiabaticity generated dissipative effect causes the generation of a dust acoustic shock wave having oscillatory behavior on the downstream side. Numerical investigations reveal that the propagation of a large amplitude dust acoustic shock wave with dust density enhancement may occur only for Mach numbers lying between a minimum and a maximum value whose dependence on the dusty plasma parameters is presented.

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I. INTRODUCTION

Experimental and theoretical investigations on low and very low frequency collective oscillations in a plasma containing micrometer sized charged dust grains—so-called ''dust ion acoustic'' and ''dust acoustic'' waves have currently gathered momentum [1–9] because of possible applications in space physics, astrophysics and also in many laboratory situations. Recently, dust ion acoustic (DIA) shock waves have been observed in the laboratory in unmagnetized dusty plasma [10,11]. The experimental findings are compared with theoretical results assuming that shock wave propagation is described by KdV Burger equation. The justification of the choice of the KdV Burger equation as a viable technique for description of dispersive shock waves was considered much earlier [12]. A dispersive shock wave is generated in a plasma when wave breaking due to the nonlinearity is balanced by the combined action of dispersion and dissipation. In absence of dissipation balancing by dispersive effect leads to the generation of solitons described by KdV equation. On the other hand, when dissipation dominates, the shock front exhibits monotonic transition of plasma density, while the shock transition is of oscillatory nature when the dissipation is weak. Dissipation is often caused by viscosity and is taken into account by the Burger term in the KdV Burger equation [13]. Landau damping and particle reflection may also cause dissipation leading to the generation of the so-called collisionless shock waves with oscillatory shock structure. One of the purposes of this paper is to show that when viscosity or Landau damping effects are not important in a dusty plasma, the nonadiabaticity of the dust charge variation provides an alternate physical mechanism causing dissipation and as a consequence this gives rise to shocks for which both monotonic and oscillatory structures are possible. It is also seen that such shocks are described by the KdV Burger equation and thus justifies the assumption made for the agreement of theoretical results with experimental findings [11].

Due to their very large inertia, the dust grains do not participate in the motion in the case of dust ion acoustic waves which have higher frequency compared to that of dust acoustic waves. As a result, the dust grain density is scarcely affected in dust ion acoustic shock waves [14–16]. Dust grains respond only to the very low frequency dust acoustic (DA) waves. The variation of dust density resulting from propagation of compressional dust acoustic shock waves is, however, of significant importance in an astrophysical context. Condensation of dust density resulting from propagation of dust acoustic shock wave through dust molecular clouds and the consequent enhanced gravitational interaction is considered as a viable process for star formation [17,18]. A similar conclusion is also suggested by the fact that the increase in dust density and the associated decrease in dust charge has the consequence that the critical length for Jean's instability is lowered [19,20]. Here we have addressed the problem of propagation of compressional dust acoustic shock waves resulting from nonadiabatic dust charge variation. To this end some introductory comments on the role of dust charge variation and its different possible approximations on the dynamics of dust acoustic waves are in order.

The charge Q_d on the dust grain is an extra dynamical variable, which controls the grain motion but itself is to be determined from the grain charging equation

$$\frac{dQ_d}{dt}=I_e+I_i=I, \tag{1}$$

where I_e and I_i are the plasma electron and ion current flowing to the dust surface. Defining hydrodynamic time scale by $\tau_d=\omega_{pd}^{-1}$ where ω_{pd} is the dust plasma frequency and the grain charging time scale by $\tau_{\rm ch}=\nu_d^{-1}$ where ν_d is the grain charging frequency, Eq. (1) can be expressed as

*Present address: Department of Applied Mathematics, University of Calcutta, Calcutta 700 009, India.

$$\frac{dQ_d}{d\left(\frac{t}{\tau_d}\right)}=\left(\frac{\tau_d}{\tau_{\rm ch}}\right)\frac{I}{\nu_d}. \tag{2}$$

For $\tau_d/\tau_{\rm ch}=\nu_d/\omega_{pd}\approx 0$, one may put $dQ_d/dt\approx 0$ yielding $Q_d=\text{const}=Q_{d0}$. In this approximation the plasma is effectively a three component plasma—one positive (ion) component and two negative ones, viz., the electrons and the micrometer sized heavy dust grains raised to a constant negative charge Q_{d0} by impinging plasma currents.

Under the opposite extreme approximation, i.e., for $\tau_d \gg \tau_{\rm ch}$ ($\omega_{pd}/\nu_d\approx 0$) the dust charge variation on the hydrodynamic time scale may be neglected and the dust charging equation (1) reduces to

$$I_e+I_i=0. \tag{3}$$

Since the electron and ion currents flowing to the dust grain surface depend both on the local electrostatic potential $\phi(x,t)$ and the grain charge Q_d [see Eqs. (11)–(14)], Eq. (2) describing ''adiabatic variation'' of the dust charge, gives the latter as a function of $\phi(x,t)$. The local equilibrium value of Q_d obtained by the assumption of adiabatic variation of dust charge provides an approximate description of the charge state of a dust grain in a truly dusty plasma. The actual grain charge is, however, a dynamical variable to be determined self-consistently from the charging equation (1) coupled to the fluid equations and Poisson's equation. The local equilibrium value or the adiabatic state dust charge given by Eq. (2) is an approximation to the actual dust charge for $\omega_{pd}/\nu_d\approx 0$.

Nonlinear propagation of dust acoustic waves have been investigated by many authors in both extreme cases $\nu_d/\omega_{pd}\approx 0$ [21–23] under different conditions and also for $\omega_{pd}/\nu_d\approx 0$ [24–27]. Among them Rao and Shukla [28] first incorporated the dust charge variations in the nonlinear theory. In these analyses, it was shown that nonlinear dust acoustic waves form solitons described by the KdV equation. Propagation of large amplitude solitary dust acoustic waves has also been studied by the pseudopotential method. The results in the two extreme cases differ only in respect of the magnitude of amplitude and velocity of the solitary waves.

The scenario changes drastically as we have shown in this paper when nonadiabaticity of the dust charge variation is taken into account by inclusion of the effect of dust charge variation on hydrodynamic time scale through replacement of Eq. (2) by $(I_e+I_i)/\nu_d=(\omega_{pd}/\nu_d)[dQ_d/d(t/\tau_d)]$. In this paper, we study the nonlinear propagation of dust acoustic waves for small ω_{pd}/ν_d both analytically for small amplitude waves and numerically for large amplitude waves. The effect of nonzero ω_{pd}/ν_d has recently been considered [26] but the result was obtained using only a linear approximation of dust charging equation. We have, however, systematically retained nonlinear contributions from Eq. (1) to appropriate higher orders. Employing the reductive perturbation technique with the scaling $\omega_{pd}/\nu_d=O(\sqrt{\epsilon})$ where ϵ is the usual expansion parameter, it is seen that nonadiabatic dust charge variation following from Eq. (1) plays a dissipative role within consequence of which the nonlinear propagation of small but finite amplitude dust acoustic wave is seen to be governed by the KdV Burger equation. The presence of the Burger term prevents any disturbance from developing into solitons and instead leads to the formation of a shocklike structure on the downstream side exhibiting either monotonic or oscillatory behavior. The generated dust acoustic shock is a compressional one providing sufficient dust density enhancement which is a prerequisite for star formation through subsequent gravitational contraction. The dust wave electrostatic potential becomes negative, and the negatively charged dust grain is raised to higher energy state as the wave passes through it.

Another effect of the Burger term is that the amplitude and velocity of an initial soliton structure decay algebraically with time. The soliton ''mass'' $\int_{-\infty}^{+\infty}\phi dx$ (ϕ is the wave amplitude), however, remains conserved leading to the generation of the so-called ''noise tail'' as the initial structure decays [29]. On the other hand, in the opposite extreme approximation, i.e., for a small nonzero value of ν_d/ω_{pd} the dust acoustic wave equation is governed by the KdV equation with a linear damping term. The soliton ''mass'' is not conserved, it decays exponentially and no shocklike structure develops.

The paper is organized in the following manner. Section II contains the basic equations. The KdV Burger equation describing the propagation is derived in Sec. III. In Sec. IV we present the results obtained from the KdV Burger description of the small but finite amplitude dust acoustic waves. Propagation of large amplitude wave is considered in Sec. V. It is found that nonadiabaticity of the dust charge variation (nonzero $\tau_{\rm ch}/\nu_d=\omega_{pd}/\nu_d$) leads to the generation of the dust acoustic shock wave when the Mach number lies between a dusty plasma parameter dependent minimum and maximum value. This has been shown by numerical integration of the equations of motion of the dust fluid and the Poisson's equation is coupled to the dust charge variation. Finally, a summary of the results is presented in Sec. VI.

II. BASIC EQUATIONS

The space and time coordinates (x,t), the grain number density, velocity charge (n_d,ν_d,Q_d), and the electric potential $\phi(x,t)$ are nondimensionalized by the substitutions

$$X=\frac{x}{\lambda_D},\quad T=\omega_{pd}t,\quad N=\frac{n_d}{n_{d0}},\quad V=\frac{\nu_d}{c_d},$$

$$Q=\frac{Q_d}{z_d e},\quad \Phi=\frac{e\phi}{T_e}, \tag{4}$$

$$\omega_{pd}=\sqrt{\frac{z_d^2e^2n_{d0}}{\epsilon_0 m_d}},\quad \lambda_D=\sqrt{\frac{\epsilon_0 T_e}{e^2\left(n_{e0}+\frac{n_{i0}}{\sigma}\right)}},$$

$$\sigma=\frac{T_e}{T_i},\quad c_d=\sqrt{\frac{z_dT_e\alpha_d}{m_d}},\quad \alpha_d=\frac{z_dn_{d0}}{\left(n_{e0}+\frac{n_{i0}}{\sigma}\right)}. \tag{5}$$

The charge on the dust grains surface is $-z_d e$ at $x=-\infty$ where the plasma is assumed to be in the undisturbed uniform state $\phi=0, n_e=n_{e0}, n_i=n_{i0}, n_d=n_{d0}$ so that

$$n_{i0}=n_{e0}+z_d n_{d0}. \tag{6}$$

On the slow time scale, the electrons and ions are in local thermodynamic equilibrium, their densities are

$$n_e=n_{e0}\exp(\Phi); \quad n_i=n_{i0}\exp\left(-\frac{\Phi}{\sigma}\right). \tag{7}$$

In terms of the nondimensionalized variables the dust fluid equations are

$$\frac{\partial N}{\partial T}+\frac{\partial (NV)}{\partial X}=0, \tag{8}$$

$$\frac{\partial V}{\partial T}+V\frac{\partial V}{\partial X}=-\frac{Q}{\alpha_d}\frac{\partial \Phi}{\partial X}. \tag{9}$$

Poisson's equation takes the form

$$\frac{\partial^2 \Phi}{\partial X^2}=-[n_{i0}e^{-\Phi/\sigma}-n_{e0}e^{\Phi}+z_d n_{d0}QN] \tag{10}$$

and the grain charging equation is

$$\left(\frac{\omega_{pd}}{\nu_d}\right)\left(\frac{\partial Q}{\partial T}+V\frac{\partial Q}{\partial X}\right)=\frac{1}{\nu_d}\frac{(I_e+I_i)}{z_d e}, \tag{11}$$

where I_e and I_i the plasma electron and ion currents flowing to the dust grain surface are given by [30–32].

$$I_e=-\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}\exp(\Phi+zQ), \tag{12}$$

$$I_i=\pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_{i0}\left(1-\frac{z}{\sigma}Q\right)\exp\left(-\frac{\Phi}{\sigma}\right), \tag{13}$$

$$z=\frac{z_d e^2}{4\pi\epsilon_0 a T_e}. \tag{14}$$

The grain charging frequency [33]

$$\nu_d=\left.\frac{\partial (I_e+I_i)}{\partial Q_d}\right|_{Q_d=-z_d e}=\frac{a}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{thi}}(1+\sigma+z). \tag{15}$$

The ion electron density ratio is obtained in terms of the plasma parameters by equating to zero the total current I_e+I_i at $x=-\infty$ where the plasma is in undisturbed state with $\phi=0$ and dust charge

$$Q_d=Q_{d0}=-z_d e \tag{16}$$

giving

$$\delta=\frac{n_{i0}}{n_{e0}}=\frac{\sqrt{\sigma}}{(\sigma+z)}\sqrt{\frac{m_i}{m_e}}e^{-z}. \tag{17}$$

Equations (6)–(17) represent all the basic equations and inter connections between the plasma parameters which govern the propagation of dust acoustic waves.

III. NONLINEAR WAVE PROPAGATION EQUATIONS

A. KdV Burger equation (nonadiabatic charge variation)

It is assumed that ω_{pd}/ν_d is small but finite

$$\frac{\omega_{pd}}{\nu_d}=\nu\sqrt{\epsilon}, \tag{18}$$

where ϵ is small and v is of order of unity and apply reductive perturbation method with stretching

$$\xi=\sqrt{\epsilon}(X-\lambda T); \quad \tau=\epsilon^{3/2}T. \tag{19}$$

The dust dynamical variables $N,V,Q=Q_d/z_d e$ and the potential Φ are expanded as

$$\begin{aligned} N&=1+\epsilon N^{(1)}+\epsilon^2 N^{(2)}+\cdots,\\ V&=\epsilon V^{(1)}+\epsilon^2 V^{(2)}+\cdots,\\ Q&=-1+\epsilon Q^{(1)}+\epsilon^2 Q^{(2)}+\cdots,\\ \Phi&=\epsilon\Phi^{(1)}+\epsilon^2\Phi^{(2)}+\cdots. \end{aligned} \tag{20}$$

Substituting the expressions for Q and Φ from Eq. (20) in Eqs. (12) and (13) and using the ordering equation (18) the dust grain charging equation (11) takes the following form in terms of the stretched variables:

$$\epsilon^2\nu\lambda\frac{\partial Q^{(1)}}{\partial \xi}=\epsilon(\beta_d\Phi^{(1)}+Q^{(1)})+\epsilon^2\left[\beta_d\Phi^{(2)}+\frac{1}{2}\frac{z\beta_d^2}{(\sigma+z)(1+\sigma+z)}\Phi^{(1)^2}+Q^{(2)}\right], \tag{21}$$

where

$$\beta_d=\frac{(\sigma+z)(1+\sigma)}{z\sigma(1+z+\sigma)}. \tag{22}$$

Expressions for dust grain charge $Q^{(1)}$ and $Q^{(2)}$ of the order of ϵ and ϵ^2 now follow:

$$Q^{(1)}=-\beta_d\Phi^{(1)}, \tag{23}$$

$$Q^{(2)}=-\beta_d\Phi^{(2)}-\frac{1}{2}\frac{z\beta_d^2}{(\sigma+z)(1+\sigma+z)}\Phi^{(1)^2}-\nu\beta_d\lambda\frac{\partial\Phi^{(1)}}{\partial\xi}. \tag{24}$$

The standard procedures applied to the dynamical equations (8),(9) and Poisson's equation (10) yield the following relations between the dependent variables of the order of ϵ and of ϵ^2:

$$N^{(1)}-\frac{V^{(1)}}{\lambda}=0, \tag{25}$$

$$V^{(1)}+\frac{\Phi^{(1)}}{\alpha_d\lambda}=0, \tag{26}$$

$$\Phi^{(1)}+\alpha_d(N^{(1)}-Q^{(1)})=0, \tag{27}$$

$$\frac{\partial N^{(1)}}{\partial\tau}+\frac{\partial(N^{(1)}V^{(1)})}{\partial\xi}=\lambda\frac{\partial N^{(2)}}{\partial\xi}-\frac{\partial V^{(2)}}{\partial\xi}, \tag{28}$$

$$\frac{\partial V^{(1)}}{\partial\tau}+V^{(1)}\frac{\partial V^{(1)}}{\partial\xi}+\frac{1}{\alpha_d}Q^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}=\lambda\frac{\partial V^{(2)}}{\partial\xi}+\frac{1}{\alpha_d}\frac{\partial\Phi^{(2)}}{\partial\xi}, \tag{29}$$

$$\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}=\alpha_dN^{(2)}-\left[\frac{\beta_d}{\lambda^2}+\frac{\left(\frac{\delta}{\sigma^2}-1\right)}{2\left(\frac{\delta}{\sigma}+1\right)}\right]\Phi^{(1)^2}-\alpha_dQ^{(2)}+(1+\alpha_d\beta_d)\Phi^{(2)}. \tag{30}$$

λ for dust acoustic waves follow from Eqs. (23) and (25)–(27)

$$\lambda=\frac{1}{\sqrt{1+\alpha_d\beta_d}}, \tag{31}$$

where $\alpha_d=(\delta-1)/[(\delta/\sigma)+1]$ by Eqs. (4), (5) and β_d is given by Eq. (22).

Next on eliminating the second order quantities $Q^{(2)}$, $N^{(2)}$, and $\Phi^{(2)}$ from Eq. (30) with the help of Eqs. (24), (28), and (29), we obtain the KdV Burger equation describing the propagation of nonlinear dust acoustic waves for $\omega_{pd}/\nu_d=O(\sqrt{\epsilon})$

$$\frac{\partial\Phi^{(1)}}{\partial\tau}-\alpha\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}+\beta\frac{\partial^3\Phi^{(1)}}{\partial\xi^3}=\mu\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}, \tag{32}$$

where

$$\alpha=\frac{\lambda^3}{2}\left[\frac{3}{\alpha_d\lambda^2}+\frac{z\alpha_d\beta_d^2}{(\sigma+z)(1+\sigma+z)}-\frac{\left(\frac{\delta}{\sigma^2}-1\right)}{\left(\frac{\delta}{\sigma}+1\right)}\right], \tag{33}$$

$$\beta=\frac{\lambda^3}{2},\quad \mu=\nu\frac{\lambda^2(1-\lambda^2)}{2}. \tag{34}$$

B. Damped KdV equation

If it is assumed that ν_d/ω_{pd} is small but finite, i.e.,

$$\frac{\nu_d}{\omega_{pd}}=\nu\epsilon^{3/2} \tag{35}$$

then the situation is opposite to that discussed in Sec. III A. The scaling (35) applied to the grain charging equation (11) leads to

$$Q^{(1)}=0,\quad \frac{\partial Q^{(2)}}{\partial\xi}=\left(1+\frac{1}{\sigma}\right)\Phi^{(1)} \tag{36}$$

while the set of relations (25)–(27) remain unchanged. Consequent to the vanishing of $Q^{(1)}$ the (normalized) phase velocity turns out to unity. After some algebra one finds that the propagation is in this case governed by the KdV equation with a damping term

$$\frac{\partial\Phi^{(1)}}{\partial\tau}-l\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}+\frac{1}{2}\frac{\partial^3\Phi^{(1)}}{\partial\xi^3}+\gamma\Phi^{(1)}=0, \tag{37}$$

where

$$l=\left[\frac{3}{2\alpha_d}-\frac{(\delta-1)}{2\left(\frac{\delta}{\sigma}+1\right)}\right],\quad \gamma=\left(1+\frac{1}{\sigma}\right)\frac{\alpha_d}{2}. \tag{38}$$

IV. CONSERVATION OF MASS AND SHOCK STRUCTURE

A. Soliton decay: Noise self

It is easily seen from the KdV Burger equation (32) that the soliton mass remains conserved

$$\frac{\partial}{\partial\tau}\int\Phi^{(1)}d\xi=0. \tag{39}$$

On the other hand,

$$\frac{\partial}{\partial\tau}\int\frac{1}{2}\Phi^{(1)^2}d\xi=-\mu\int\left(\frac{\partial\Phi^{(1)}}{\partial\xi}\right)^2d\xi. \tag{40}$$

The method of Karpman and Maslov yields the approximate solution

$$\Phi^{(1)}(\xi,\tau)=A(\tau)\mathrm{sech}^2\sqrt{\frac{\alpha A(\tau)}{12\beta}}\left[\xi-\frac{\alpha}{3}\int_0^\tau A(y)dy\right], \tag{41}$$

where

$$A(\tau)=\frac{A_0}{\left(1+\frac{4A_0\alpha\mu\tau}{45\beta}\right)}. \tag{42}$$

The dissipation causes the soliton amplitude to decay algebraically and transfer to so-called ‘‘noise tail’’ [29]

$$\Phi_{\mathrm{tail}}=\left(\frac{4\mu}{15}\right)\sqrt{\frac{A_0}{\left(1+\frac{4A_0\alpha\mu\tau}{45\beta}\right)}} \tag{43}$$

and thus allow for the conservation of soliton mass.

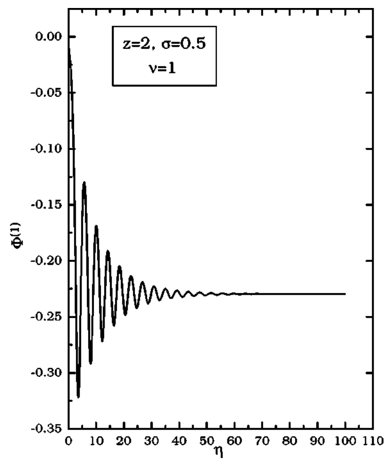

FIG. 1. Oscillatory shock structure for weak dissipation. ν defined by Eq. (18).

B. Shock structure: Energization of dust particles

The Burger term in Eq. (32) implies the possibility of existence of the shocklike structure. On transforming to the wave frame

$$\eta = V\tau - \xi = \sqrt{\epsilon}\frac{[c_d(\lambda + \epsilon V)t - x]}{\lambda_D} \tag{44}$$

the KdV Burger equation (32) reduces to

$$\frac{d^2\Phi^{(1)}}{d\eta^2} = \frac{V}{\beta}\Phi^{(1)} + \frac{\alpha}{\beta}\Phi^{(1)^2} - \frac{\mu}{\beta}\frac{d\Phi^{(1)}}{d\eta}. \tag{45}$$

Equation (45) has two fixed points ($\Phi^{(1)}=0, d\Phi^{(1)}/d\eta=0$) and ($\Phi^{(1)}=-2V/\alpha, d\Phi^{(1)}/d\eta=0$). The first one $\Phi^{(1)}=0$ is a saddle point while the second one, viz., $\Phi^{(1)}=-2V/\alpha$ is a stable focus or a stable node according as

$$M > \text{or} < 1 + \frac{\omega_{pd}^2(1-\lambda^2)^2}{\nu_d}, \tag{46}$$

where M is defined by the ratio of the nonlinear wave velocity to the linear dust acoustic wave velocity $c_d\lambda$

$$M = 1 + \epsilon\frac{V}{\lambda}. \tag{47}$$

The relation (46) is defined by using Eqs. (18) and (47).

$\Phi^{(1)}(\eta)$ is obtained by numerical integration of Eq. (45) subject to the boundary conditions $\Phi^{(1)} \rightarrow 0$ at $\eta \rightarrow -\infty$. Thus for any value of x the potential builds up from near zero value at long past $t \rightarrow -\infty(\eta \rightarrow -\infty)$ to a steady value

$$\Phi^{(1)} = -2(M-1)\frac{\lambda T_e}{e\alpha} \tag{48}$$

as $t \rightarrow \infty$ shows shock-wave-like structure as illustrated in Figs. 1 and 2 with oscillating transition corresponding to stable focus at the second fixed point. Since the potential is negative, the negatively charged dust is energized by the passing wave to

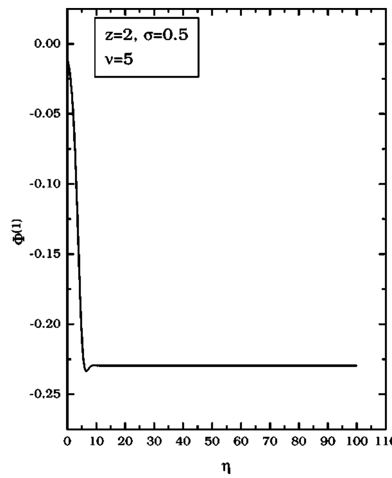

FIG. 2. Monotonic shock structure for strong dissipation. ν defined by Eq. (18).

$$E = -z_d e\phi = 2(M-1)\frac{\lambda z_d T_e}{\alpha}. \tag{49}$$

For hydrozen ion dusty plasma $E/(M-1)z_d e T_e$ is plotted in Fig. 3 against $\delta = n_{i0}/n_{e0}$ for fixed $\sigma = T_i/T_e$. The ratio increases with δ until it reaches a maximum and then goes down. However, it is to be noted according to Eq. (17) that with σ fixed, δ increases with decrease in z which for given z_d and T_e is achieved only by increases in the grain radius a.

V. LARGE AMPLITUDE DUST ACOUSTIC SHOCK

Assume that the dust fluid has a nonzero flow velocity far upstream. The upstream boundary conditions on the normalized variables are

$$N=1, \quad V=V_{dr}, \quad \Phi=0, \quad Q=-1. \tag{50}$$

Nonzero dust drift velocity V_{dr} causes modification [30] of the expression (11) for the plasma ion current to the dust grain surface by terms $O(V_{dr}/V_{thi})$ where V_{thi} is the ion

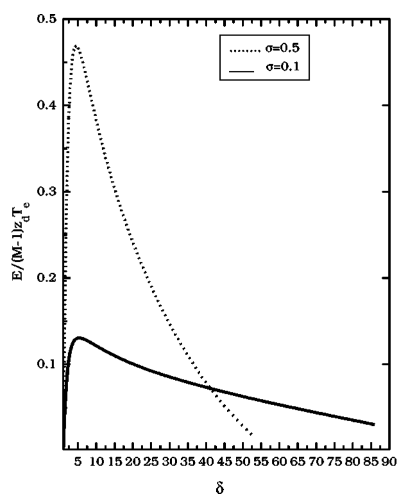

FIG. 3. $E=$energy [as given by Eq. (49)] to which dust grains are raised by passing shock waves.

thermal velocity. We neglect such contributions assuming $V_{dr} \ll V_{thi}$. Transforming to the frame of the wave with wave velocity λ

$$\zeta = X - \lambda T \tag{51}$$

and using boundary conditions (50), Eq. (8) is at once integrated to yield

$$N(V-\lambda) = u, \tag{52}$$

where

$$u = V_{dr} - \lambda. \tag{53}$$

With V given by Eq. (52) the charging equation (11) in the wave frame (51) becomes

$$\left(\frac{\omega_{pd}}{\nu_d}\right)\left(\frac{u}{N}\right)\frac{d\Delta Q}{d\xi} = \frac{1}{\nu_d}\frac{(I_e+I_i)}{z_d e}. \tag{54}$$

Set

$$Q = -1 + \Delta Q \tag{55}$$

and substitute for I_e, I_i, and ν_d. Equation (54) now takes the form

$$f(\Phi,\Delta Q) + \left(\frac{\omega_{pd}}{\nu_d}\right) g\left(\Phi,\Delta Q,\frac{d\Delta Q}{d\zeta}\right) = 0, \tag{56}$$

where

$$f(\Phi,\Delta Q) = \exp(\Phi + z\Delta Q) - \left[1 - \frac{z}{(z+\sigma)}\Delta Q\right]\exp\left(-\frac{\Phi}{\sigma}\right), \tag{57}$$

$$g\left(\Phi,\Delta Q,\frac{d\Delta Q}{d\zeta}\right) = \frac{z(1+\sigma+z)}{(\sigma+z)}\frac{u}{N}\frac{d\Delta Q}{d\zeta}. \tag{58}$$

For ω_{pd}/ν_d small, ΔQ is obtained from Eq. (57) by successive approximation. To $O(\omega_{pd}/\nu_d)$ we have

$$\Delta Q = F(\Phi) - \left(\frac{\omega_{pd}}{\nu_d}\right)\frac{u}{N}\exp\left(\frac{\Phi}{\sigma}\right)\frac{F(\Phi)\dfrac{d\Phi}{d\zeta}}{[1-\chi F(\Phi)]}. \tag{59}$$

The derivation of the above approximation for ΔQ and the explicit expression for $F(\Phi)$ are given in the Appendix.

On eliminating V from Eq. (9) with the help of Eq. (52), the equation of motion for the dust fluid reduces to

$$\frac{dN}{d\zeta} = \frac{1}{\alpha_d}\left(\frac{N^3}{u^2}\right)[-1+\Delta Q]\frac{d\Phi}{d\zeta}. \tag{60}$$

Poisson's equation (10) is rewritten as

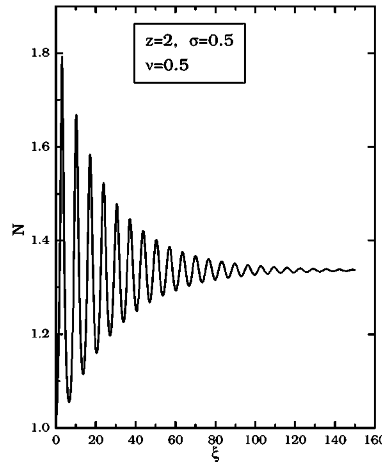

FIG. 4. Oscillatory nature of large amplitude shock wave governed by Eqs. (60), (61). N denotes the dust number density. $\nu = \omega_{pd}/\nu_d$.

$$\frac{d^2\Phi}{d\zeta^2} = -\frac{\sigma}{\delta}\left[\delta\exp\left(-\frac{\Phi}{\sigma}\right) - \exp(\Phi) + (\delta-1)N(-1+\Delta Q)\right]. \tag{61}$$

Equating Eqs. (60) and (61) with ΔQ given by Eq. (59) form a closed system. Since the right hand side of Eqs. (60) and (61) are free from explicit ζ dependence it is permissible to choose $\zeta=0$ as the upstream point for purpose of numerical integration and integrate up to large positive values of ζ. Starting from a small perturbation of the boundary conditions (50) and upon numerical integration of the above system of equations by Runge-Kutta-Fehlberg method of order 5, it is seen that the perturbation develops into a shock wave provided the dust velocity V_{dr} far upstream exceeds the phase velocity λ of the wave. Prototype of the dust shock wave structure is shown in Fig. 4. The transition from the far upstream value to the far downstream value may occur with an oscillating behavior.

Investigation based on numerical integration of Eqs. (55) and (61) keeping δ and σ fixed but varying u reveals certain feature of the shock propagation. It is seen that shock wave is generated only for $u = V_{dr} - \lambda$ lying between two extreme values

$$0 < u_{\min}(\delta,\sigma) \leq u \leq u_{\max}(\delta,\sigma). \tag{62}$$

The dependence of $u_{\min}$ and $u_{\max}$ on δ, for different σ, was calculated numerically and is demonstrated graphically in Fig. 5.

For adiabatic dust charge variation $(\omega_{pd}/\nu_d = 0)$ existence of dust acoustic solitary waves becomes possible provided the pseudopotential satisfies certain conditions [24,25]. Such conditions lead to imposition of bounds on the solitary wave velocity and hence on the wave Mach number. The dust density tends to become infinite as the limiting values of the Mach number is approached. The dust density N associated with dust acoustic shocks, governed by Eqs. (60) and

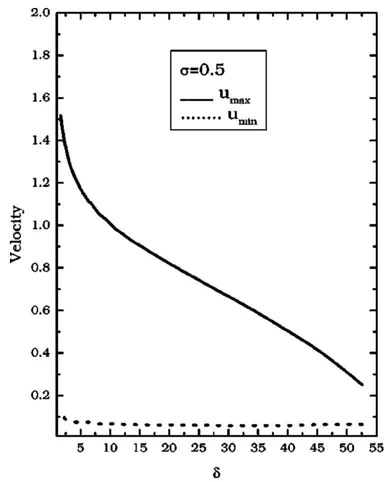

FIG. 5. Variation of $u_{\max}$ and $u_{\min}$ with (σ, δ). Large amplitude shock wave generation occurs only for $u_{\min}<U=V_{dr}-\lambda<u_{\max}$.

(61) shows a similar behavior. But $u_{\min}$ or $u_{\max}$ cannot be determined from the analytical conditions on pseudopotential as in case of solitary waves simply because no pseupotential can exist when nonadiabaticity of dust charge variation is taken into account. The limits $u_{\min}$ and $u_{\max}$ are determined by varying u and integrating Eqs. (60) and (61) numerically until $|N|\to\infty$. The graphical behavior shown in Fig. 5 gives the limiting values approximately. Figure 6 shows the corresponding bounds of the Mach number M defined by

$$M=\frac{V_{dr}}{\lambda}, \tag{63}$$

$$M_{\min}=1+\frac{u_{\min}}{\lambda}, \quad M_{\max}=1+\frac{u_{\max}}{\lambda}. \tag{64}$$

VI. DISCUSSION

Discussions involving a summary of the results is presented in this section. (a) It is shown that the nonadiabatic variation of dust charge causes dissipation represented by the term $\mu(\partial^2\Phi^{(1)}/\partial\xi^2)$ in the KdV Burger equation (32) describing small amplitude dust acoustic shock. It is a collisionless shock in the sense that no viscous or damping effects resulting from collisions between dust and plasma particles are involved. It is a new physical mechanism entirely different from that involved in generating the shock described by KdV Burger equation for DIA shock wave [11]. The dissipation coefficient μ is proportional to $\omega_{pd}/\nu_d=\tau_{\rm ch}/\tau_d$ vanishes in the adiabatic limit, i.e., hydrodynamic time scale $\tau_d\gg\tau_{\rm ch}$, charging time.

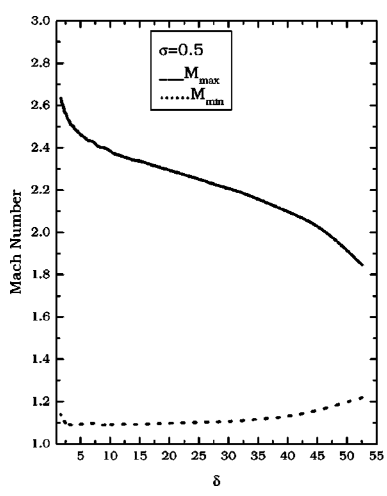

FIG. 6. Variation of $M_{\max}$ and $M_{\min}$ with (σ, δ). Large amplitude shock wave generated by Eqs. (60), (61) only for $M_{\min}<M<M_{\max}$.

(b) Steady large amplitude shock wave propagation is described by coupled equations (60) and (61) for dust density N and electrostatic potential Φ. The effect of nonadiabaticity of dust charge variation is represented by the term proportional to ω_{pd}/ν_d in Eq. (59).

(c) The structure of steady small amplitude shocks given by Eq. (45) are shown in Figs. 1, 2. The transition from upstream to far downstream state changes from being of oscillatory to monotonic nature as dissipation μ increases.

(d) Figure 3 shows oscillatory transition of dust density N from upstream normalized unperturbed value (unity) to higher value far downstream for a large amplitude shock.

(e) Numerical investigation of Eqs. (60) and (61) shows that large amplitude shocks can occur only for upstream dust drift velocity $V_{\rm dr}=\lambda+u$ lying between two extreme limits $\lambda+u_{\min}$ and $\lambda+u_{\max}$ with corresponding Mach number M satisfying $M_{\min}<M<M_{\max}$ [Eqs. (62)–(64)]. The critical Mach numbers $M_{\min}$ and $M_{\max}$ are functions of $\delta=n_{i0}/n_{e0}$ and $\sigma=T_i/T_e$. The dependence of $u_{\min}$, $u_{\max}$ and $M_{\min}$, $M_{\max}$ are plotted in Fig. 5 and Fig. 6, respectively. The dust density N shows rapid increase with M and dust condensations becomes very intense as $M\to M_{\max}$ (in the numerical integration $N\to\infty$ as $M\to M_{\max}$). According to the current theory [18] such dust acoustic shock induced intense dust condensation in interstellar dust cloud may suffice to initiate gravitional contraction leading to star formation.

(f) Figure 3 shows the plot of the energy E (with suitable normalization) to which the negatively charged dust grains are raised by the negative potential of the passing electrostatic shock.

(g) Approximate analytic solution of the KdV Burger equation shows that an initial solitary wave structure decays algebraically as shown by Eqs. (41) and (42). The decay is associated with development of a noise tail in consequence of soliton mass conservation [Eq. (39)].

APPENDIX

Let ΔQ be approximately given by

$$\Delta Q=F(\Phi)+\frac{\omega_{pd}}{\nu_d}G\left(\Phi,\frac{d\Phi}{d\xi}\right) \tag{A1}$$

for small ω_{pd}/ν_d. Substituting in Eq. (51) and Taylor expanding to $O(\omega_{pd}/\nu_d)$

$$f[\Phi,\Delta Q=F(\Phi)]+\frac{\omega_{pd}}{\nu_d}\frac{\partial f}{\partial \Delta Q}\bigg|_{\Delta Q=F(\Phi)}G\left(\Phi,\frac{d\Phi}{d\xi}\right)+\frac{\omega_{pd}}{\nu_d}g\left(\Phi,F(\Phi),\frac{d\Phi}{d\xi}\right)=0. \tag{A2}$$

Equating to zero terms independent of ω_{pd}/ν_d and $O(\omega_{pd}/\nu_d)$,

$$f(\Phi,\Delta Q)=0, \tag{A3}$$

$$G\left(\Phi,\frac{d\Phi}{d\xi}\right)=-\left[\frac{\partial f}{\partial \Delta Q}\bigg|_{\Delta Q=F(\Phi)}\right]^{-1}g\left(\Phi,F(\Phi),\frac{d\Phi}{d\xi}\right). \tag{A4}$$

By taking logarithm, transform equation (A3) to the following form:

$$\frac{(\sigma+1)}{\sigma}\Phi+z\Delta Q=\ln(1-p\Delta Q);\quad \rho=\frac{z}{(z+\sigma)} \tag{A5}$$

and expand the solution in a power series of Φ

$$\Delta Q=F(\Phi)=\sum_k a_k\Phi^k. \tag{A6}$$

Substituting the power series in Eq. (A5) and equating coefficients of different powers of Φ, the a_k's are obtained recursively

$$a_1=-\frac{(1+\sigma)(\sigma+z)}{z\sigma(1+\sigma+z)},$$

$$a_2=-\frac{a_1^2\rho^2}{2(\rho+z)},$$

$$a_3=-\frac{\rho^2}{(\rho+z)}\left[a_1a_2+\rho\frac{a_1^3}{3}\right],$$

$$a_4=-\frac{\rho^2}{(\rho+z)}\left[a_1a_3+\frac{a_2^2}{2}+\rho a_1^2a_2+\rho^2\frac{a_1^4}{4}\right],$$

$$a_5=-\frac{\rho^2}{(\rho+z)}\left[a_1a_4+a_2a_3+\rho(a_1^2a_3+a_1a_2^2)+\rho^2a_1^3a_2+\rho^3\frac{a_1^5}{5}\right],$$

$$a_6=-\frac{\rho^2}{(\rho+z)}\left[a_1a_5+\frac{a_3^2}{2}+\rho\left(2a_1a_2a_3+a_1^2a_4+\frac{a_2^2}{3}\right)+\rho^2\left(a_1^3a_3+3a_2^2\frac{a_1^2}{2}\right)+\rho^3a_1^4a_2+\rho^4\frac{a_1^6}{6}\right]. \tag{A7}$$

It is found that Eqs. (A6) and (A7) give $F(\Phi)$.

For the range of values of Φ in the present calculation, it is sufficient to evaluate the series in Eq. (A6) to terms of degree six in Φ giving $F(\Phi)$ to error <0.01 when compared to the exact value of ΔQ given by $f(\Phi,\Delta Q)=0$.

Differentiating $f(\Phi)$ with respect to ΔQ, using Eq. (A3), $G(\Phi,d\Phi/d\xi)$ given by Eq. (A4) simplifies to

$$G\left(\Phi,\frac{d\Phi}{d\xi}\right)=-\frac{u}{N}\exp\left(\frac{\Phi}{\sigma}\right)\frac{F(\Phi)}{[1-\chi F(\Phi)]}\frac{d\Phi}{d\xi} \tag{A8}$$

after some algebra, Eqs. (1) and (8) now give the expression (59).

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